How To Study Maths Effectively?

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Dear students,

It is my great pleasure to interact with you. As a teacher, it is my main desire to help you realise, the great potential each one of you have. I have been training young students for several years. Every year a number of my students are selected at the State and National level Olympiad examinations and also at IIT Entrance examination. Some of these have won medals even at the International Mathematical Olympiad. Today, I wish to share with you some of the effective methods of studying. (Mathematics in particular.)

Deceptive Success

Most of the students have been brain washed to believe that, their main aim is to score maximum marks in an examination. So, such students memorise the answers to number of questions and do get very high percentage of marks in the board type examinations. Do you know what happens to them subsequently? It is well known that the cut-off level, for engineering admission (in a decent college) is above 90%. Last year about 16,000 students appeared for the First Year Engineering Examination. Only 4,000 managed to pass. Among those who passed only 400 secured first class.

Reasons of failure

Let us look at the nature of the 10th board question paper. Most of the questions are already known to the students as these are directly chosen from the 10th textbook. Moreover huge option is available. Known nature of the questions allows the students to score undeserved marks by merely memorising the answers without any understanding of the subject. The available 'option' gives students the liberty to avoid D-level (tough) problems. Choice of questions from only 10th textbook gives them the license to forget most of the concepts learnt in the earlier years. In spite of these unhealthy methods of studying, students do score very high percentage of marks. Unfortunately their faith in mindless mugging gets strengthened. 12^{th} board examination being of the similar nature their success story continues. Finally and inevitably they meet their Waterloo at the first year engineering examination. If you do not learn from the failures of these seniors, I am afraid that you are heading to be the next *Abhimanyu*. (*Arjuna's* son in *Mahabharata*)

Let us understand the difference between Information and Knowledge.

Let me share with you an anecdote. When I was is in 10^{th} standard my Physics teacher wrote down three Newton's laws of motion on the blackboard. He made us memorise these. Once he was satisfied that each one of us had correctly remembered these laws, he asked us “My Dear Students what have you learnt today?" All of us very proudly told him that we had learnt Newton's Laws. He laughed loudly. He then told us that we had merely memorised the statements of the Newton's Laws. He then spent next couple of months in explaining the meaning of each and every word in these statements and making us apply these in solving a number of practical problems. I am indebted to my teacher for bringing out the difference between information and the knowledge.

Let us have a close look at a typical student from standard VII. He computes simple interest using the formula $I = (P \times R \times N) / 100$. During the course of the year he solves more than 50 similar problems. Let us carefully observe what he is actually doing. He is merely computing the product of P, R, and N and dividing it by 100. He then checks this number with the number given at the back of the book. If it does not tally, he computes once again. Once his computations agree with the given answer, he solves one more similar problem, repeating this procedure all over again. All these efforts only ensure that the student has correct information about how to compute simple interest. To gain the knowledge he must understand why this formula really computes the simple interest. The proof, the derivation of the formula is really what mathematics is all about. If a student is not in a position to prove formula for the simple interest, do you think that he will be able to prove the formula for calculating the amount in the case of compound interest?

Check for yourself in the following situations whether you have only information or have the knowledge also.

(1) Heron's formula for calculating the area of the triangle.
(2) Rule for divisibility of a natural number by 9.
(3) Formula for the area of a trapezium.
(4) The method of constructing angle bisector of a given angle.
(5) The rules of Logarithm.

(6) The ratio of the circumference of a circle to its diameter is same for all circles.

Lokamanya Tilak had given us the slogan:
"Freedom is my birthright and I will achieve it."

On the same lines I want you to take the oath:
Knowledge is my birthright and I will acquire it. I will not accept any formula without proof; I will learn the justification of each and every geometric construction.

To implement this oath successfully you need to learn more about your most important instrument the brain.

Understanding the way the brain works

Let me give you an analogy. I have more than 2000 books in my personal library. If I buy a new book and arbitrarily place it somewhere then it will be very difficult for me to retrieve the book. I should certainly not keep a maths book along with the history books. Among the maths books, 1 must separately store the books on algebra geometry, arithmetic, calculus etc. Among the geometry books, I should further classify according Euclidean Geometry, Co-ordinate Geometry, Differential Geometry, and Algebraic Geometry and so on. In our brain we have an astonishing amount of information stored. So any new concept we learn we must be extremely careful in keeping it in the correct place in the brain. Now how do we really do it? If you study a new idea in isolation then in a very short time you will forget it. You must make a conscious effort to associate the new idea with a number of similar ideas already existing in the brain.

Whenever you study a new theorem follow as many of the following suggestions as possible.

(1) First try to think about how the discoverer of this result might have found out this result. What experiments, he must have done which could have led him to this result.
(2) Try to prove this result independently. Who knows, your solution may be a much simpler one!
(3) If you do not get the proof, just read a couple of lines of the proof and with this starting point try once again.
(4) Once the complete proof is understood list all the theorems, which you have used in proving this theorem. Prove each of these.
(5) Find out number of corollaries of this theorem.
(6) Find the various ways in which this result can be generalised.

(7) Teach this theorem to your friends. All these steps will ensure that you have placed this new theorem in a correct place inside your brain.

Now you are bound to ask me "How will we find so much time to do all these things?" Let us return to the example of simple interest. Instead of wasting time in mechanically solving around 100 problems, learn to derive the formula. You will be surprised to discover that its proof is so simple. Having mastered the proof you won't find any need to solve more than 5 to 10 typical problems. Now you still have a lot of time, which you could utilise to understand the concept of compound interest and derive the formula for amount in this case. You could create a number of formulae in the case of compound interest according as the interest is paid after each month or each day or even after each hour. You could relate this concept to population growth problem and estimate the future population of our country. Hey! Probably you have already started enjoying mathematics. Having mastered the topic you could now very quickly scan through all the problems in me text book and identify that actually then are only 4 to 6 type of different problems in this chapter. This ability of sifting the essence from the gross is the first indicator of your future success.

What is the next step?

In most of the schools, under the pretext of the revision the teachers will ask you to solve these 100 problems all over again. Solving these problems all over again is essentially going to be an exercise to the hand and not to the brain. Rebel! Refuse, politely but firmly, to do this mindless homework, though India is a free country, you must earn your own freedom.

Discover the creativity within yourself. Try to construct some new problems on the topic you have completed from the school textbook. Hold a competition among your friends and give a prize to the best discovery of the week. Go beyond school textbooks; Join the Library of *Bhaskaracharya Pratishsthana* or British Council Library. Subscribe to some journals like *Samasya*, *Resonance*. Participate in the Olympiad Movement.

Reference Books

Here is a list of some of really wonderful books you should include in your personal library.

(1) Mathematical Circles: A Russian Experience. Author: Fomin. I am particularly fond of this book. (Rs. 195)

(2) Challenge and thrill of pre college mathematics. Author: Krishnamurty. (Rs. 260)
(3) An Excursion in Mathematics. Published by *Bhaskaracharya Pratishsthana* (Rs. 65)
(4) Adventures in Problem Solving. Author Shailesh Shirali. (Rs. 200) He has written many books. All are very good.
(5) Elementary Number Theory Author: David Burton. (Rs. 175)
(6) *Aisee Prameye Rasike* (in Marathi) Author: Ravindra Bapat (Rs. 175)

Here is a list of some world famous authors. Try to read as many books written by them. (a) Yaglom (b) Polya (c) Coxeter (d) Ross Honsberger (e) Sharygin (f) Martin Gardner.

All these books an available at Universal Bookstall 216, Narayan Peth Pune 411030. Phone Number: (020)24451780, (020)24450976

Email: universalbooks@eth.net

I sincerely hope that you will put to use various ideas I have discussed here. I am sure that you will start enjoying studying as never before. Wish you all the best in your exploration of this beautiful universe.

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